Structural Analysis by Hand

VBCOA – Region 5
May 15, 2014

Presenter

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Logistics

Exits

Restrooms

Cell Phones

No Smoking

NOTICE

NO SMOKING
UNLESS YOU’RE ON FIRE
Questions?

SURE—Go ahead and ask your question!

This class is interactive!

There are no stupid questions!

Attendees

- Inspector
- Plan reviewer
- Contractor
- Engineer
- Architect
- Designer
- Suppliers
- Attorney

Objectives

- Determine loading requirements for joists and beams
- Apply loads using free body diagrams
- Calculate moment and deflection
- Analyze compliance for flexure, shear and deflection
- Analyze simple steel beam
- Analyze spread footings
**Assumptions**
- No structural theory
- Knowledge of arithmetic and basic algebra
- Simple loading
  - Uniform loads
  - Point loads at midspan
- Simple spans
- Wood, LVL, steel
- Residential construction

**Math 101**
3. Find $x$.

**Operation Sequence**
- Please Excuse My Dear Aunt Sally
  - Parenthesis
  - Exponent
  - Multiplication
  - Division
  - Addition
  - Subtraction
P.E.M.D.A.S.

\[ 5^2 + (3 - 1) / 2 - 4 \times 3 \]

Step 1: Parenthesis \[5^2 + 2 / 2 - 4 \times 3\]
Step 2: Exponents \[25 + 2 / 2 - 4 \times 3\]
Step 3: Multiply \[25 + 2 / 2 - 12\]
Step 4: Divide \[25 + 1 - 12\]
Step 5: Add \[26 - 12\]
Step 6: Subtract \[14\]

Mathematical Formulas

- Volume of a box
- Length \( \times \) Width \( \times \) Height
- Variables
  - \( L \) = Length
  - \( W \) = Width
  - \( H \) = Height
  - \( V \) = Volume
- \( V = L \times W \times H \)
- \( V = LWH \)

Mathematical Formulas

- \( V = LWH \)
- Values
  - \( L = 5' \)
  - \( W = 2' \)
  - \( H = 6' \)
- \( V = 5 \times 2 \times 6 \)
- \( V = (5)(2)(6) \)
- \( V = 60 \text{ feet}^3 \)
You Try It, Find V

- $V = \frac{1}{3} lwh$
- Values:
  - $l = 2'$
  - $w = 3'$
  - $h = 6''$
- Convert $h$ value from inches to feet: $6'' = 0.5'$
- Insert values into formula:
  $$V = \frac{1}{3} (2)(3)(0.5) = 1 \text{ ft}^3$$

A Word About Units

- Always include units with every calculation
- Ensure all calculations are completed in the same units (inches, feet)
**Vertical Load Path**

Vertical load path transfers gravity load (snow):
- to roof sheathing
- to rafters
- to top plate
- to studs
- to bottom plate
- to foundation & footings
- to ground

**Design Methodologies**

**Load & Resistance Factor Design (LRFD)**
- Applied loads adjusted up
- Resistance capacity of structural member adjusted down
- Compare values: capacity > loads

**Allowable Stress Design (ASD)**
- Actual stress calculated using applied loads
- Structural member’s allowable stresses calculated
- Compare values: allowable > actual

**Joist/Beam Analysis**

"You have clearly been under enormous stress."
Sample First Floor Framing Plan

Step 1: Determine uniform dead load
- Units for uniform dead load
  - pounds per square foot
  - lbs/ft²
  - psf
- Dead Load: weight of structure
  - Assume 10 PSF for floors
  - Assume 15 PSF for roofs

Step 2: Determine uniform live load (psf)
- Live Load: weight produced by use and occupancy
  - People
  - Furniture
  - Vehicles
- Units: pound per square foot (psf)
  - IBC Table 1607.1
  - IRC Table R301.5
Step 2: Determine uniform live load (psf)

- For floors: IRC Table R301.5

<table>
<thead>
<tr>
<th>Category</th>
<th>Live Load (psf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Offices without storage</td>
<td>15</td>
</tr>
<tr>
<td>Offices with limited storage</td>
<td>20</td>
</tr>
<tr>
<td>Computer centers</td>
<td>40</td>
</tr>
<tr>
<td>Reception areas</td>
<td>30</td>
</tr>
<tr>
<td>Restrooms</td>
<td>40</td>
</tr>
<tr>
<td>Utility rooms and offices served with heat caps</td>
<td>30</td>
</tr>
<tr>
<td>Dormitories</td>
<td>40</td>
</tr>
<tr>
<td>Uncovered and handrails</td>
<td>20</td>
</tr>
<tr>
<td>Hardwood floors</td>
<td>20</td>
</tr>
<tr>
<td>Premium vehicle garages</td>
<td>30</td>
</tr>
<tr>
<td>Roof over sleeping room</td>
<td>40</td>
</tr>
<tr>
<td>Storage rooms</td>
<td>30</td>
</tr>
<tr>
<td>Ceilings</td>
<td>40</td>
</tr>
</tbody>
</table>

Step 2: Determine uniform live load (psf)

- For roofs, greatest of live load or snow load
- Snow load: IRC Section R301.2.3
  - Northern Virginia counties 20 – 30 psf

Step 2: Determine uniform live load (psf)

- Roof live load: IRC Table R301.6

<table>
<thead>
<tr>
<th>Roof Slope</th>
<th>Tributary Loaded Area in Square Feet for Any Structural Member</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat or rise less than 4 inches per foot (1:3)</td>
<td>20</td>
</tr>
<tr>
<td>Rise 4 inches per foot (1:3) to less than 12 inches per foot (1:1)</td>
<td>16</td>
</tr>
<tr>
<td>Rise 12 inches per foot (1:1) and greater</td>
<td>12</td>
</tr>
</tbody>
</table>
Step 3: Calculate tributary width (feet)

- Load influence distance from each side of a framing member
  - **Joists**: half the distance to next adjacent joists on each side
  - **Beams**: half of the joists’ span that bear on each side of the beam

\[
TW = \frac{16}{2} + \frac{16}{2} = 16" = 1.33' \\
\text{Convert to feet} = \frac{16}{12} = 1.33' 
\]

**EXAMPLE:** JOIST

**EXAMPLE:** BEAM

12'-0"
4'-0"
10'-0"
**Step 3: Calculate tributary width (feet)**

**EXAMPLE:** BEAM

\[ TW = \frac{4}{2} + \frac{10}{2} = 7\text{'} \]

**Step 4: Calculate linear load (plf)**

- \( w = \text{uniform load} \times TW \)
- Units: pounds per linear foot = lbs/ft = plf
- **EXAMPLE:**
  - JOIST: \( w_{LL} = (40)(1.33) = 53.33 \text{ plf} \)
  - \( w_{DL} = (10)(1.33) = 13.33 \text{ plf} \)
  - \( w = 66.67 \text{ plf} \)
  - BEAM: \( w_{LL} = (40)(7) = 280.0 \text{ plf} \)
  - \( w_{DL} = (10)(7) = 70.0 \text{ plf} \)
  - \( w = 350.0 \text{ plf} \)

**Free Body Diagram**

- A two-dimensional graphic symbolization of a structural member which models bearing locations and loading elements
- Linear load, \( w: \)
  - 100 plf
- Point load, \( P: \)
  - 500 lbs
- Bearing locations (reaction), \( R: \)
Free Body Diagram

Joist

Beam

Example: Free Body Diagram

On plans provided, first floor joist adjacent fireplace hearth extension:

You Try It

Draw the free body diagram of the sunroom beam

Show load and its value
Total uniform load = 50 psf
Tributary width = 5’
Total linear load = (50)(5) = 250 plf
Show span length
Step 5: Bending Analysis

- Flexure, bending, moment, torque
- Highest at midspan for uniform load

Pulling stress or tension on bottom face of member

Step 5A: Determine $F'_b$ (psi)

- Allowable bending stress, $F'_b$
- The maximum bending stress permissible for a specified structural member
- Units for stress:
  - pounds per square inch
  - lbs/in^2
  - psi

Step 5A: Determine $F'_b$ (psi)

- “Raw” value based on wood species: $F_b$
- Adjusted allowable bending stress,
  $F'_b = F_b C_M C_F C_r C_D$

  - $C_M =$ Service condition (wet or dry)
  - $C_D =$ Load duration (normal or snow)
  - $C_r =$ Repetitive use (joists, 3+ply beams)
  - $C_F =$ Member size (2x?)
Step 5A: Determine \( F'_b \) (psi)

- Use tables in chart based on:
  - Species
  - Service condition (wet or dry)
  - Load duration (normal or snow)
  - Single (2-ply beam) or repetitive (joists)
  - Member size

**EXAMPLE:**
- Joists: Repetitive, dry, 2x8, normal load duration, Hem-Fir#2: \( F'_b = 1,173 \) psi
- Beam: Single, dry, (2)1¾x9½, normal load duration, Microlam: \( F'_b = 2,684 \) psi

Step 5B: Determine \( b \) (in), \( d \) (in), \( S \) (in³)

Section Modulus:
\[
S = \frac{1}{6} bd^2
\]

**EXAMPLE:**
- JOIST: \( S = \frac{1}{6} (1.5)(7.25)^2 = 13.1 \text{ in}^3 \)
- BEAM: \( S = (2) \left( \frac{1}{6} \right) (1.75)(9.5)^2 = 52.6 \text{ in}^3 \)

Step 5B: Determine \( S \) (in³)

**EXAMPLE:**
- JOIST: \( S = 13.1 \text{ in}^3 \)
- BEAM: \( S = (2)(26.3) = 52.6 \text{ in}^3 \)
Step 5C: Determine Span Length, l (ft)
Calculate Moment, M (lbs-ft)

\[ M = \frac{wl^2}{8} \]

where:
- \( l \) = span length, ft
- \( w \) = total linear load, plf
- \( M \) = moment, lbs-ft

EXAMPLE:

JOIST: \( w = 66.67 \) plf, \( l = 10' \)
\[ M = \frac{(66.67)(10)^2}{8} = 833.4 \text{ lbs-ft} \]

BEAM: \( w = 350 \) plf, \( l = 12' \)
\[ M = \frac{(350)(12)^2}{8} = 6,300 \text{ lbs-ft} \]

Step 5D: Calculate \( f_b \) (psi)

- Actual bending stress, \( f_b \)
- The bending stress a specified structural member is experiencing under maximum applied load

\[ f_b = \frac{12M}{S} \]

where:
- \( S \) = section modulus, in\(^3\)
- \( M \) = moment, lbs-ft
- \( f_b \) = actual bending stress, psi
Step 5D: Calculate $f_b$ (psi)

$$f_b = \frac{12M}{S}$$

where:
- $S$ = section modulus, in$^3$
- $M$ = moment, lbs-ft
- $f_b$ = actual bending stress, psi

**EXAMPLE:**

**JOIST:**
- $M = 833.4$ lbs-ft
- $S = 13.1$ in$^3$
- $f_b = \frac{(12)(833.4)}{13.1} = 763.4$ psi

**BEAM:**
- $M = 6,300$ lbs-ft
- $S = 52.6$ in$^3$
- $f_b = \frac{(12)(6,300)}{52.6} = 1,437.3$ psi

Step 5E: Compare $F'_b$ with $f_b$

If $f_b \leq F'_b$, then member is **good** for bending

**EXAMPLE:**

**JOIST:**
- $f_b = 763.4$ psi < $F'_b = 1,173$ psi  **OK!**

**BEAM:**
- $f_b = 1,437.3$ psi < $F'_b = 2,684$ psi  **OK!**

Step 6: Shear Analysis

- Shear is similar to a cutting stress
- Highest at ends = reaction
- Wood shear analysis uses shear value at a distance from the end equal to member's depth

Member experiences a slicing action in vertical plane
**Step 6A: Determine $F'_v$ (psi)**

- **Allowable shear stress, $F'_v$**
- The maximum shear stress permissible for a specified structural member
- Units for stress:
  - pounds per square inch
  - lbs/in²
  - psi

** EXAMPLE:**
- Joist: Dry, Hem-Fir#2: $F'_v = 150$ psi
- Beam: Dry, Microlam: $F'_v = 285$ psi
**Step 6B: Calculate Area, A (in²)**

Area:

\[ A = bd \]

**EXAMPLE:**

JOIST: \( A = (1.5)(7.25) = 10.9 \text{ in}^2 \)

BEAM: \( A = (2)(1.75)(9.5) = 33.2 \text{ in}^2 \)

---

**Step 6B: Calculate Area, A (in²)**

<table>
<thead>
<tr>
<th>MEMBER</th>
<th>SECTION PROPERTIES</th>
<th>( A_{in^2} )</th>
<th>( S_{in^4} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2x4</td>
<td>1.5 3.5 3.5 3.5 3.5</td>
<td>3.06 3.06 3.06</td>
<td>3.06 3.06 3.06</td>
</tr>
<tr>
<td>2x6</td>
<td>1.5 3.5 3.5 3.5 3.5</td>
<td>3.15 3.15 3.15</td>
<td>3.15 3.15 3.15</td>
</tr>
<tr>
<td>2x8</td>
<td>1.5 3.5 3.5 3.5 3.5</td>
<td>3.24 3.24 3.24</td>
<td>3.24 3.24 3.24</td>
</tr>
<tr>
<td>2x10</td>
<td>1.5 3.5 3.5 3.5 3.5</td>
<td>3.33 3.33 3.33</td>
<td>3.33 3.33 3.33</td>
</tr>
<tr>
<td>2x12</td>
<td>1.5 3.5 3.5 3.5 3.5</td>
<td>3.42 3.42 3.42</td>
<td>3.42 3.42 3.42</td>
</tr>
<tr>
<td>2x16</td>
<td>1.75 3.5 3.5 3.5 3.5</td>
<td>3.51 3.51 3.51</td>
<td>3.51 3.51 3.51</td>
</tr>
<tr>
<td>2x20</td>
<td>1.75 3.5 3.5 3.5 3.5</td>
<td>3.60 3.60 3.60</td>
<td>3.60 3.60 3.60</td>
</tr>
<tr>
<td>2x24</td>
<td>1.75 3.5 3.5 3.5 3.5</td>
<td>3.69 3.69 3.69</td>
<td>3.69 3.69 3.69</td>
</tr>
<tr>
<td>2x30</td>
<td>1.75 3.5 3.5 3.5 3.5</td>
<td>3.78 3.78 3.78</td>
<td>3.78 3.78 3.78</td>
</tr>
<tr>
<td>2x36</td>
<td>1.75 3.5 3.5 3.5 3.5</td>
<td>3.87 3.87 3.87</td>
<td>3.87 3.87 3.87</td>
</tr>
</tbody>
</table>

**EXAMPLE:**

JOIST: \( A = 10.9 \text{ in}^2 \)

BEAM: \( S = (2)(16.6) = 33.2 \text{ in}^2 \)

---

**Step 6C: Calculate Shear, V (lbs)**

\[ V = w \left( \frac{l}{2} - \frac{d}{12} \right) \]

where:

- \( l \) = span length, ft
- \( w \) = total linear load, plf
- \( d \) = member depth, in

**EXAMPLE:**

JOIST: \( w = 66.67 \text{ plf} \)

\[ l = 10' \]
\[ d = 7.25" \]

\[ V = 66.67 \left( \frac{10}{2} - \frac{7.25}{12} \right) = 293.0 \text{ lbs} \]

BEAM: \( w = 350 \text{ plf} \)

\[ l = 12' \]
\[ d = 9.5" \]

\[ V = 350 \left( \frac{12}{2} - \frac{9.5}{12} \right) = 1822.9 \text{ lbs} \]
Step 6D: Calculate $f_v$ (psi)

- Actual shear stress, $f_v$
- The shear stress a specified structural member is experiencing under maximum applied load

$$f_v = \frac{3V}{2A}$$

where:
- $f_v$ = actual shear stress, psi
- $A$ = area, in$^2$
- $V$ = shear, lbs

**EXAMPLE:**

JOIST: $V = 293.0$ lbs
$A = 10.9$ in$^2$
$$f_v = \frac{3(293.0)}{2(10.9)} = 40.4 \text{ psi}$$

BEAM: $V = 1,822.9$ lbs
$A = 33.2$ in$^2$
$$f_v = \frac{3(1,822.9)}{2(33.2)} = 82.4 \text{ psi}$$

Step 6E: Compare $F'_v$ with $f_v$

If $f_v \leq F'_v$, then member is good for shear

**EXAMPLE:**

JOIST: $f_v = 40.4$ psi $< F'_v = 150$ psi **OK**
BEAM: $f_v = 82.4$ psi $< F'_v = 285$ psi **OK**
Step 7: Deflection Analysis

- "Sag" a member experiences
- Analysis is based on live load only
- Highest at midspan

![Deflection Diagram]

Step 7A: Determine allowable deflection (in)

- Use Table R301.7

<table>
<thead>
<tr>
<th>Structural Member</th>
<th>Allowable Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rafter having slope greater than 3:12 with no finished ceiling attached to rafter</td>
<td>1/36</td>
</tr>
<tr>
<td>Interior walls and partitions</td>
<td>1/180</td>
</tr>
<tr>
<td>Floors and slabs with finish or ceiling finish</td>
<td>1/240</td>
</tr>
<tr>
<td>All other structural members</td>
<td>1/200</td>
</tr>
<tr>
<td>Exterior walls—wind loads* with finish or ceiling finish</td>
<td>1/180</td>
</tr>
<tr>
<td>Exterior walls with other finish designs</td>
<td>1/120</td>
</tr>
<tr>
<td>Lintel—supporting masonry veneer walls*</td>
<td>2/600</td>
</tr>
</tbody>
</table>

where \( l \) = span length of member, in.

(span length must be converted from feet to inches)

Step 7B: Calculate allowable deflection (in)

EXAMPLE: Floor, \( \Delta_{\text{max}} = \frac{12l}{360} \)

JOIST: \( \Delta_{\text{max}} = \frac{(12)(10)}{360} = 0.33'' \)

BEAM: \( \Delta_{\text{max}} = \frac{(12)(12)}{360} = 0.40'' \)
Step 7C: Calculate $I$ (in$^4$)

**Moment of Inertia:**

$$I = \frac{1}{12} bd^3$$

**EXAMPLE:**

JOIST: $I = \frac{1}{12} (1.5)(7.25)^3 = 47.6$ in$^4$

BEAM: $I = (2) \left( \frac{1}{12} \right) (1.75)(9.5)^3 = 250$ in$^4$

---

Step 7C: Calculate $I$ (in$^4$)

**EXAMPLE:**

JOIST: $I = 47.6$ in$^4$

BEAM: $I = (2)(125) = 250$ in$^4$

---

Step 7D: Determine $E$ (psi)

- **Modulus of elasticity, $E$**
- The mathematical description of a structural member’s elastic characteristics
- **Units for modulus of elasticity:**
  - pounds per square inch
  - lbs/in$^2$
  - psi
**Step 7D: Determine E (psi)**

- **Modulus of elasticity**
- “Raw” value based on wood species: \( E \)
- Adjusted modulus of elasticity: \( E' \)
  
  \[ E' = E \cdot C_M \]

  where:
  
  - \( C_M \) = Service condition (wet or dry)

**Step 7E: Calculate actual deflection (in)**

\[
\Delta_{act} = \frac{22.5 \cdot w_{LL} \cdot l^4}{EI}
\]

where:

- \( \Delta_{act} \) = actual deflection
- \( l \) = span length, ft
- \( w_{LL} \) = linear live load, plf
- \( E \) = modulus of elasticity, psi
- \( I \) = moment of inertia, in\(^4\)
Step 7E: Calculate actual deflection, \( \Delta_{\text{act}} \) (in)

EXAMPLE:

JOIST: \( w_{\text{LL}} = 53.3 \text{ plf} \)
\( l = 10' \)
\( E = 1.3 \times 10^6 \text{ psi} \)
\( I = 47.6 \text{ in}^4 \)

\[ \Delta_{\text{act}} = \frac{(22.5)(53.3)(10)^4}{(1.3 \times 10^6)(47.6)} = 0.194'' \]

BEAM: \( w_{\text{LL}} = 280 \text{ plf} \)
\( l = 12' \)
\( E = 1.9 \times 10^6 \text{ psi} \)
\( I = 250 \text{ in}^4 \)

\[ \Delta_{\text{act}} = \frac{(22.5)(280)(12)^4}{(1.9 \times 10^6)(250)} = 0.275'' \]

Step 7F: Compare \( \Delta_{\text{max}} \) to \( \Delta_{\text{act}} \)

If \( \Delta_{\text{act}} \leq \Delta_{\text{max}} \), then member is good for deflection.

EXAMPLE:

Joist: \( \Delta_{\text{act}} = 0.194'' < \Delta_{\text{max}} = 0.333'' \) OK!

Beam: \( \Delta_{\text{act}} = 0.275'' < \Delta_{\text{max}} = 0.400'' \) OK!

JOIST PASSES!

BEAM PASSES!

When good beams go bad...

- When one component fails, the structural member fails
- RULE OF THUMB: the bending stress is the gauge
- In a uniformly loaded member, shear will pass if flexure passes
- Deflection is critical when flexure barely passes
**Shortcuts...“Dirty numbers”**

- Span length: no need to be exact
- More than one live load on a member?
  - Use value of live load member mostly sees, or
  - use highest value
- Round values up
- Drop decimals

**Examples – Go to the Plans**

**Example: Garage Rafters**

- Go to Sheet S4 on the provided plans
- Do rafters comply?
Example: Garage Rafters

- **Step 1**: Determine uniform dead load, $DL = 15 \text{ psf}$
- **Step 2**: Determine uniform live load, $LL = 20 \text{ psf}$
- **Step 3**: Determine tributary area, $TW = 2'$
- **Step 4**: Calculate linear load
  - $w_{DL} = (2)(15) = 30 \text{ plf}$
  - $w_{LL} = (2)(20) = 40 \text{ plf}$
  - $w = 70 \text{ plf}$
- **Step 5A**: Determine $F'_{b}$
  - Repetitive, dry, snow load duration, SPF So. #2
  - $F'_{b} = 1,230 \text{ psi}$

Example: Garage Rafters

- **Step 5B**: Determine $S$
  - For 2x8, from chart, $S = 13.1 \text{ in}^2$
- **Step 5C**: Determine $l$, calculate $M$
  - $l = 11'$
  - $w = 70 \text{ plf}$
  - $M = \frac{(70)(11)^2}{8} = 1,059 \text{ lbs ft}$
- **Step 5D**: Calculate $f_b = \frac{(12)(1,059)}{13.1} = 967 \text{ psi}$

Example: Garage Rafters

- **Step 5E**: Compare
  - $f_b = 967 \text{ psi} < F'_{b} = 1,230 \text{ psi}$
  - Flexure analysis **OK**!
- **Step 6A**: Determine $F'_{v}$
  - Dry, SPF So. #2
  - $F'_{v} = 135 \text{ psi}$
- **Step 6B**: Determine $A$ (in$^2$)
  - For 2x8, from chart, $A = 10.9 \text{ in}^2$
- **Step 6C**: Calculate $V = 70 \left( \frac{11}{2} \times \frac{7.25}{12} \right) = 343 \text{ lbs}$
**Example: Garage Rafters**

- **Step 6D:** Calculate $f_v = \frac{(3)(343)}{(2)(10.9)} = 47.2$ psi
- **Step 6E:** Compare:
  - $f_v = 47.2$ psi < $F_v' = 135$ psi
  - Shear analysis **OK!**
- **Step 7A:** Determine $\Delta_{\text{max}}$
  - Roof, slope > 3:12, no ceiling finish
  - $l/180$
- **Step 7B:** Calculate
  
  $\Delta_{\text{act}} = \frac{(12)(11)(5.22)}{(343)(3)} = 0.733''$

**Example: Garage Rafters**

- **Step 7C:** Determine $I$
  - For 2x8, from chart, $I = 47.6$ in$^4$
- **Step 7D:** Determine $E'$
  - Dry, SPF So. #2
  - $E' = 1.1 \times 10^6$ psi
- **Step 7E:** Calculate
  
  $\Delta_{\text{act}} = \frac{(22.5)(40)(11)}{(1.1 \times 10^6)(47.6)} = 0.25''$

**Step 7F:** Compare:
  - $\Delta_{\text{act}} = 0.25'' < \Delta_{\text{max}} = 0.733''$
  - Deflection analysis **OK!**

**RAFTER PASSES!**

**Example: Second Floor Header**

- Go to Sheet S3 on the provided plans
- Does header comply?
Example: Second Floor Header

- **Step 1:** Determine uniform dead load, DL = 10 psf
- **Step 2:** Determine uniform live load, LL = 30 psf
- **Step 3:** Determine tributary area, TW = 14’
- **Step 4:** Calculate linear load
  - \( w_{DL} = (14)(10) = 140 \text{ plf} \)
  - \( w_{LL} = (14)(30) = 420 \text{ plf} \)
  - \( w = 560 \text{ plf} \)
- **Step 5A:** Determine \( F'_{b} \)
  - Repetitive, dry, normal duration, SPF So. #2
  - \( F'_{b} = 891 \text{ psi} \)

Example: Second Floor Header

- **Step 5B:** Determine \( S \)
  - For 2x12, from chart, \( S = (3)(31.6) = 94.8 \text{ in}^3 \)
- **Step 5C:** Determine \( I \), calculate \( M \)
  - \( I = 10’ \)
  - \( w = 560 \text{ plf} \)
  - \( M = \frac{(560)(10)^2}{8} = 7,000 \text{ lbs} \cdot \text{ft} \)
- **Step 5D:** Calculate \( f_h = \frac{(12)(7,000)}{94.8} = 886 \text{ psi} \)

Example: Second Floor Header

- **Step 5E:** Compare:
  - \( f_h = 886 \text{ psi} < F'_{b} = 891 \text{ psi} \)
  - Flexure analysis **OK!**
- **Step 6A:** Determine \( F'_{v} \)
  - Dry, SPF So. #2
  - \( F'_{v} = 135 \text{ psi} \)
- **Step 6B:** Determine \( A \)
  - For 2x12, from chart, \( A = (3)(16.9) = 50.7 \text{ in}^2 \)
- **Step 6C:** Calculate \( V = 560 \left( \frac{10}{2} - \frac{11.25}{12} \right) = 2,275 \text{ lbs} \)
Example: Second Floor Header

- Step 6D: Calculate $f_v = \frac{(3)(2.275)}{(2)(50.7)} = 67$ psi
- Step 6E: Compare:
  - $f_v = 67$ psi < $F'v = 135$ psi
  - Shear analysis OK!
- Step 7A: Determine $\Delta_{max}$
  - Floor
  - $l/360$
- Step 7B: Calculate $\Delta_{max} = \frac{(12)(10)}{360} = 0.333"$

Example: Second Floor Header

- Step 7C: Determine $l$
  - For 2x12, from chart, $l = (3)(178) = 534$ in$^4$
- Step 7D: Determine $E'$
  - Dry, SPF So. #2
  - $E' = 1.1x10^6$ psi
- Step 7E: Calculate $\Delta_{act} = \frac{(22.5)(420)(10)^3}{(1.1x10^6)(534)} = 0.16"$
- Step 7F: Compare:
  - $\Delta_{act} = 0.16" < \Delta_{max} = 0.333"$
  - Deflection analysis OK!

HEADER PASSES!

You Try It

- Using the house plans provided, do the sunroom joists and beam comply?
### JOISTS

1. Uniform dead load: $DL = 10$ psf
2. Uniform live load: $LL = 40$ psf
3. Tributary width: $TW = 2'$
4. Linear load: $w_{DL} = 2 \times 10 = 20$ plf  
   $w_{LL} = 2 \times 40 = 80$ plf  
   $w = 100$ plf
5A. Allowable bending stress: $F'_b = 1,070$ psi
5B. Section modulus: $S = 13.1$ in$^3$
5C. Span length, moment: $l = 10'$
   $$M = \frac{(100)(10)^2}{8} = 1,250 \text{ lbs} - \text{ft}$$

### JOISTS

5D. Actual bending stress:
   $$f_b = \frac{(12)(1,250)}{13.1} = 1,141.6 \text{ psi}$$
5E. Compare: $f_b = 1,141.6$ psi > $F'_b = 1070.0$ psi  **NG**

**JOIST FAILS!**

### BEAM

1. Uniform dead load: $DL = 10$ psf
2. Uniform live load: $LL = 40$ psf
3. Tributary width: $TW = 5'$
4. Linear load: $w_{DL} = 5 \times 10 = 50$ plf  
   $w_{LL} = 5 \times 40 = 200$ plf  
   $w = 250$ plf
5A. Allowable bending stress: $F'_b = 1,070$ psi
5B. Section modulus: $S = (3)(13.1) = 39.3$ in$^3$
5C. Span length, moment: $l = 10.33'$
   $$M = \frac{(250)(10.33)^2}{8} = 3,335 \text{ lbs} - \text{ft}$$
5D. Actual bending stress:

\[ f_b = \frac{(12)(3.335)}{39.3} = 1,016 \text{ psi} \]

5E. Compare:

\[ f_b = 1,016 \text{ psi} < F'_b = 1,070 \text{ psi} \quad \text{OK} \]

6A. Allowable shear stress: \( F'_{\nu} = 135 \text{ psi} \)

6B. Area: \( A = (3)(10.9) = 32.7 \text{ in}^2 \)

6C. Shear:

\[ V = 250 \left( \frac{10.33}{2} - \frac{7.25}{12} \right) = 1,140 \text{ lbs} \]

6D. Actual shear stress:

\[ f_{\nu} = \frac{(3)(1140)}{(2)(32.7)} = 52.5 \text{ psi} \]

6E. Compare:

\[ f_{\nu} = 52.5 \text{ psi} < F'_{\nu} = 135 \text{ psi} \quad \text{OK} \]

7A. Allowable deflection:

\[ \Delta_{\text{max}} = \frac{(12)(10.33)}{360} = 0.344\" \]

7B. Moment of inertia: \( I = (3)(47.6) = 142.8 \text{ in}^4 \)

7C. Modulus of elasticity: \( E' = 1.1 \times 10^6 \text{ psi} \)

7D. Actual deflection:

\[ \Delta_{\text{act}} = \frac{22.5(200)(10.33)}{(1.1 \times 10^6)(142.8)} = 0.326\" \]

7E. Compare:

\[ \Delta_{\text{act}} = 0.326\" < \Delta_{\text{max}} = 0.344\" \quad \text{OK} \]

**BEAM PASSES!**
Steel Beams

- Wide flange section
- A992, high strength
- A36, normal strength
- Design check for uniformly loaded beam
- Basement location
- Simple spans

Step 1: Determine Span Length (ft)

- From plans, choose longest span of a beam type, W8x18
- For example, assume 17’

Step 2: Determine Allowable Load (kips)
**Step 3: Determine Uniform Loads (psf)**

- First floor
  - Live load = 40 psf
  - Dead load = 10 psf
- Second floor
  - Live load = 30 psf (if sleeping)
  - Dead load = 10 psf

**Step 4: Determine Tributary Width (ft)**

\[ TW = \frac{13.25}{2} + \frac{12.75}{2} = 13' \]

**Step 5: Calculate Actual Linear Load (plf)**

- First floor:
  \[ w_1 = (13)(40 + 10) = 650 \text{ klf} \]
- Second floor:
  \[ w_2 = (13)(30 + 10) = 520 \text{ plf} \]
- Total:
  \[ w = 1,170 \text{ plf} \]
Step 6: Calculate Actual Total Load (kips)

- **Actual Load** = (17)(1,170) = 19,890 lbs = 19.9 kips

Step 7: Compare Actual and Allowable Loads

- Actual = 19.9 kips > Allowable = 14 kips NG!
  
  **BEAM FAILS!**
  
  Try W8x28, $L_{max} = 23$ kips > 19.9 kips, OK!

Rules of Thumb

- The deeper the beam the more it can span
- A steel beam can usually span 2 feet for every inch of depth
  - W8, span=16’
  - W10, span=20’
**You Try It**
- Go to plans
- Check steel beam in basement

**Solution**
- W8x18, span length = 12’
- Allowable load = 20 kips
- \( TW = \frac{12.75 + 15.33}{2} = 14’ \)
- Total uniform load:
  - \( w_1 = 14 \times (40 + 10) = 700 \text{ plf} \)
  - \( w_2 = 14 \times (30 + 10) = 560 \text{ plf} \)
- Actual = \( 12 \times 1,260 = 15.1 \text{ kips} < 20 \text{ kips OK!} \)

**Steel Beams**
**Spread Footings**

- Soil has maximum capacity to withstand pressure
- Footing distributes point load to soil based on maximum capacity

**Step 1: Determine Presumptive Soil Bearing Pressure, \( Q_{\text{max}} \) (psf)**

- Determine using Table R401.4.1
- Maximum assumable often identified by AHJ
- For example, \( Q_{\text{max}} = 1,500 \) psf

<table>
<thead>
<tr>
<th>CLASS OF MATERIAL</th>
<th>LOAD-BEARING PRESSURE (pounds per square foot)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crystalline bedrock</td>
<td>12,000</td>
</tr>
<tr>
<td>Sedimentary and foliated rock</td>
<td>4,000</td>
</tr>
<tr>
<td>Sandy gravel and/or gravel (GW and GIP)</td>
<td>3,000</td>
</tr>
<tr>
<td>Sand, silt, sand, sandy soil, silt, gravel and clays gravel (GW, SS, SM, SC, GM and GC)</td>
<td>2,000</td>
</tr>
<tr>
<td>Clay, sandy clay, silty clay, silty soil, silt and sandy silt (CL, ML, MH and CH)</td>
<td>1,500P</td>
</tr>
</tbody>
</table>

**How to Calculate Reactions**

\[
R = \frac{wl}{2} \quad \text{where: } l = \text{span length, ft} \quad w = \text{total linear load, plf} \quad R = \text{reaction, lbs}
\]
## Step 3: Calculate Point Load

- Point load = \( R_1 + R_2 \)
- For example:
  \[ R_1 = \frac{(1,170)(13)}{2} = 7,605 \text{ lbs} \]
  \[ R_2 = \frac{(1,170)(17)}{2} = 9,945 \text{ lbs} \]
- \( P = 17,550 \text{ lbs} \)

## Step 2: Calculate \( Q_{\text{act}} \) (psf)

- Calculate actual bearing pressure:
  \[ Q_{\text{act}} = \frac{P}{LW} \]
  - \( P \) = column load, lbs
  - \( L \) = footing length, ft
  - \( W \) = footing width, ft
- For example:
  \[ Q_{\text{act}} = \frac{17,550}{(4)(4)} = 1,097 \text{ lbs} \]

## Step 4: Compare \( Q_{\text{act}} \) to \( Q_{\text{max}} \)

- \( Q_{\text{act}} = 1,097 \text{ psf} < Q_{\text{max}} = 1,500 \text{ psf} \), OK!

**FOOTING PASSES!**
You Try It

- Go to the plans provided
- Are the spread footings supporting the steel columns adequate?

Solution

- Soil type = silty sand (SM), $Q_{\text{max}} = 2,000$ psf
- $P = \frac{(1,170)(11.33)}{2} + \frac{(1,170)(12)}{2} = 13,648$ lbs
- $L = W = 2.5'$
- $Q_{\text{act}} = \frac{13,648}{(2.5)(2.5)} = 2,184$ psf
- $Q_{\text{act}} = 2,416$ psf > $Q_{\text{max}} = 2,000$ psf; NG!

"That's all Folks!"